Epistemic Term-Modal Logic

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Abstract. In the present paper syntax, constant domain semantics with non-rigid constants and an axiom system for n agent systems with identity for a term-modal logic is presented. Examples of expressibility will be given, and venues of further research addressed.

In his 1637 *Discourse on Method*, French philosopher René Descartes famously wrote *Cogito ergo sum*: I think, therefore I am. Without dwelling in details of Descartes' writings, one may see the proposition as loosely based on existential instantiation – thinking is a property, and if something possess the property, it must, too, exist. But though a classic epistemic and existential proposition, *Cogito ergo sum* is not expressible in classic, first-order epistemic logic.

In fact, even in full-fledged second-order epistemic modal logic, there are certain such elements of first-order reasoning that cannot be expressed. These regard the reasoning agents, their role as knowers and their existence. For though our agents are able to reason both about objects and their properties and their own and other agents information about such matters, they are not able to reason about neither themselves nor other agents as objects. Had Descartes used classic first-order epistemic logic to formalize his arguments, he could never have concluded his own existence.

The reason for this is two-fold. From a semantic point of view, the agents of epistemic logic do not, strictly speaking, exist in the most basic sense, namely in the domains of the models for our various first-order modal logics. Even if we stipulate a subset of the domain to be agents and add agent predicates, this will be insufficient due to syntactic limitations: the property of being "a knower" is expressed by structurally crude modal operators. The agent-denoting indices of such operators are not terms of the first-order language, and hence these do not work as such either.

To exemplify, if we assume $P(a)$ true at some state in a model, it would not be a fair reading of $K_aP(a)$ that agent a knows of himself/agent a that he is P, for the two occurrences of 'a' is neither syntactically nor semantically connected. In fact, the formula blatantly misuses notation. This is the case as the set of operators are defined using an index set *distinct* from the terms of the first-order language – often $I = \{1, 2, ..., n\}$. Hence, we should write, for example, $K_1P(a)$ clearly marking the lack of connection between the knower, 1, and the know-of object, a.

As a result, there are certain aspects of regular first-order reasoning, like that of Descartes, that we are still not able to express in first-order modal logic.

To give an example, imagine student a having to obtain information φ regarding who is to speak after lunch at some summer school. Let us further assume that all members of the Program Committee knows φ and that this is known by a, i.e. $K_a \forall x (PC(x) \rightarrow K_x \varphi)$. Thus a could conclude that b should know φ if b is known to be an organizer. Yet, the universal implication known by a cannot be expressed in classic first-order epistemic logic – it is simply not well-formed.[1]

The lack of expressibility is due to that fact that the knowledge/belief/etc. operators "only" serve as operators – they do not, too, serve as predicates. And as predicates are what expresses properties in first-order modal logic, neither knowledge, belief or other intentional attitudes can be formalized as properties.

In the present paper syntax, constant domain semantics with non-rigid constants and an axiom system for n agent systems with identity that allow for this kind of reasoning will be presented. In order to capture the relevant aspect of agency, namely existence, agents are added explicitly to the domain and are each correlated with an accessibility relation and operators. The operators are indexed by terms of the first-order language in order to ensure that they operate as both operators and predicates. Examples of expressibility will be given and venues of further research addressed. The semantics proposed have been constructed so as to allow for the construction of *normal* term-modal logics, and a variant of the *canonical model theorem* has been proven in [10]. The term 'term-modal logic' was coined in [11], where completeness is shown for sequent calculi and tableaux systems for K, D, T, K4, D4 and S4 for semantics with monotone domains, rigid constants and function symbols and without identity. Earlier, in [9] a term-modal logic with constant domains and non-rigid constants is formulated for belief and soundness shown for KD45. Truth-value gaps are used to render $B_t \varphi$ false for non-agent denoting terms, resulting in non-standard truth-conditions for the operators, in turn resulting in the lack of validity of Dual and Knowledge Generalization. In [8] a first-order dynamic term-modal logic is constructed. Non-rigid constants are used along with a constant domain consisting only of agents, and quantification is done by wildcard assignment. In[6], Hintikka does at late passage use the operators as predicates and quantify over the indices, and the mention of the idea can be found as early as [12].

Changes to syntax

The changes to the syntax of ordinary first-order epistemic logic in order to gain the expressibility required to quantify over agents and denote these with constants are not difficult. Define a language $\mathcal{L}^{\mathbf{n}}$ for a n agent logic a countably infinite set of variables VAR, a set of constants, CON, and a set of relation symbols, REL, where both CON and REL are (possibly empty) countable.[2]

[1] The same kind of reasoning could be forced in propositional epistemic logic with an 'Everybody in G knows that' operator, E_G, as $E_G \varphi \rightarrow K_i \varphi$ is valid for all $i \in G$ – but the information whether i is in G or not is not information available to the agents as it is a meta-logical condition.

[2] The notation REL_n will be used to denote the subset of REL consisting of n-ary relation symbols.

The set TER of terms of $\mathcal{L}^{\mathbf{n}}$ are $VAR \cup CON$. The language is relativized to n agents by requiring the existence of a subset $AGT \subseteq VAR$ where $|AGT| \geq n$. The well-formed formulas of $\mathcal{L}^{\mathbf{n}}$ (henceforth formulas) are given by

$$\varphi ::= R^n (t_1, ..., t_n) \mid t_1 = t_2 \mid \neg\varphi \mid \varphi \wedge \psi \mid \forall x \varphi \mid K_t \varphi.$$

where $R^n \in REL_n$, $t, t_1, ..., t_n \in TER$ and $x \in VAR$.
The remaining logical connectives, the existential quantifier and the dual operator as well as definitions of free and bound variables and sentences are all as usual.

This use of the modal operators allow for them to serve both a role as operators, but also a role as classic predicates, hence providing us with the wished for added expressibility. To exemplify, where φ is a formula, both $\exists x K_x \varphi$ and $K_a \varphi \wedge (a = b) \rightarrow K_b \varphi$ are well-formed formulas proper.

Index set and classes of operators. In regular modal logics, the set of modal operators is, so to speak, under control. That is, it is simply defined so that for each $i \in I$ for some finite set of agents I, K_i is an operator. Given the above syntax the set of operators depend on the set of terms. Once we define the semantics we will only have a finite set of accessibility relations, and each operator is then correlated with a relation via the elements of the domain, thus creating classes of semantically equivalent operators.

There are two reasons to introduce an index set in order to partition the operators, even though it is not necessary to define the set of well-formed formulas. The first pertains to the adding of additional axioms: if there is no partition of the operators, there is not way to add axioms for a specific subset, i.e. a class of operators. We do want to be able to add class-specific axioms, though, as each class will contain exactly the operators for one specific agent. Hence adding axioms for a specific class will allow differing epistemic strength of the agents.

Secondly, a partition of the operators is very handy when proving completeness of axiom systems by canonical models, see [10]. The reason is that the classes of semantically equivalent operators are induced by the domain of the semantic structures, as defined below. But as the domain of the canonical models is normally defined through the accessibility relations (which is usually defined through inclusion of operators in maximal consistent sets), we need to "cut the loop", making sure we can define these classes by other than semantic means. To this end we introduce, for a language $\mathcal{L}^{\mathbf{n}}$, an index set $I = \{1, 2, ..., n\}$ and a surjection

$$i_{AGT} : I \longrightarrow \Phi_{AGT}$$

where Φ_{AGT} is a partition of AGT with each class being countably infinite.[3] This ensures that we will not run out of variables denoting each agent. We enumerate the class $i_{AGT}(k) \in \Phi_{AGT}$ by $\{x_{k1}, x_{k2}, ...\}$. On the basis of this partition of the agent denoting variables, we define a partition of the operators $\mathsf{K} = \{K_x \varphi : x \in VAR, \varphi \in \mathcal{L}^{\mathbf{n}}\}$ of $n + 1$ sets such that for $k = 1, ..., n$, $\mathsf{K}_k =$

[3] That is $\Phi_{AGT} = \{AGT_k : \cup_{k \in I} AGT = AGT \wedge (k \neq l \Rightarrow AGT_k \neq AGT_l) \wedge \forall k \in I : |AGT_k| = \aleph_0\}$

$\{K_{x_{km}}\varphi : x_{km} \in i_{AGT}(k)\}$ and $\mathsf{K}_A = \{K_x\varphi : \neg\exists k \in I : x \in i_{AGT}(k)\}$. We denote the partition $\{\mathsf{K}_1, ..., \mathsf{K}_n, \mathsf{K}_A\}$ by K/i, and where $K_x\varphi \in \mathsf{K}_i, i \in \{1, ..., n, A\}$, we will call K_x a K_i-modality. For each $k \in I$, the class of operators will be correlated by the semantics with one of n agent in the domain. The last class is related to all other objects.

Semantics

Frames. As we are now dealing with languages with more expressive power than standard first-order modal logic, the definition of frames becomes increasingly complex.[4] Thus items 4 and 5 below will not occur in standard definition of first-order frames. They are required for present purposes as we need to be sure this information is encoded at the most basic level of the semantic structures. Item 4 defines the set of agents included in the domain, and item 5 correlates each object of the domain with an accessibility relation.
Definition: n-frame. An **n**-*frame* for a language $\mathcal{L}^{\mathbf{n}}$ is a quintuple $\mathcal{F} = \langle W, R^{\mathbf{n}}, Dom, Agt^{\mathbf{n}}, \sim\rangle$ where

1. W is an non-empty is a set of states
2. $R^{\mathbf{n}}$ is a non-empty set of n relations on W (that is, $R^{\mathbf{n}} = \{R_1, ..., R_n\}$,and $\forall R_i \in R^{\mathbf{n}} : R_i \subseteq W \times W$)
3. Dom is a non-empty domain of quantification
4. $Agt^{\mathbf{n}}$ is a privileged subset of Dom consisting of n elements, called *agents*
5. $\sim: Dom \longrightarrow R^{\mathbf{n}} \cup \{W \times W\}$ is a function such that $\forall i \in Agt^{\mathbf{n}} :\sim (i) \in R^{\mathbf{n}}$ and $\forall j \in Dom \backslash Agt^{\mathbf{n}} :\sim (j) = W \times W$ ■

To ease notation in the following, we enumerate $Agt^{\mathbf{n}}$ by $\{a_1, ..., a_n\}$ and assume that $\sim (a_i) = R_i$. The elements of W will also be referred to as *worlds*, and below we will often omit explicit reference to **n** when speaking of **n**-frames.

Though it would be interesting to work with more general frames, with no restrictions on the domain and agent set, the present semantics are simpler, and hence easier to work with. Allowing for varying domains – and with them varying agent sets – would allow for interesting applications, like the death or firing of agents, but would complicate proving meta-theoretical results.

In 5. in the definition, each agent from $Agt^{\mathbf{n}}$ is related its accessibility relation and further, each non-agent object is related to the universal binary relation on W. The latter is partly an artificial choice as elements of the domain that are not agents should never know anything. From a mathematical point of view this is choice is the preferred one over mapping the class to $\emptyset$, as this would require a far more complex axiom system. In addition, our choice leads to operators from K_A to behave like global modalities, which adds extra expressibility to the logics and eases meta-theoretical results. Though, as formulas $K_t\varphi$ where t is a non-agent denoting term, and K_t thus a global modality, will only be true when

[4] At least the expressive power is different. It has not been proven that we have an conservative extension of the systems defined in for example [3] or [4], though this is suspected.

φ is a valid in a specific model, one can take it that this construction merely models what it is to be dumb as a door. Further, adding global modalities for all non-agent objects allows the rule of Knowledge Generalization to preserve truth and Dual to remain valid (see further below).

Interpretation, model and valuation. We now define an *interpretation* to be a function

$$\mathcal{I} : REL_n \times W \longrightarrow \mathcal{P}(Dom^n)$$

that to each n-ary relation symbol assigns some set of n-tuples $(d_1, ..., d_n)$, where $\{d_1, ..., d_n\} \subseteq Dom$ relative to each world $w \in W$. The special case of identity is treated by letting $\mathcal{I}(=, w) = \{(d, d) : d \in Dom\}$ for all w. Further, let

$$\mathcal{I} : CON \times W \longrightarrow Dom$$

and we denote the value of constant c in w with $\mathcal{I}(c, w)$ where $\mathcal{I}(c, w) \in Dom$.[5] A frame $\mathcal{F}$ augmented with an interpretation $\mathcal{I}$ is called a *model*, and is denoted $M = \langle \mathcal{F}, \mathcal{I} \rangle$, and a *valuation* is defined as a surjective (onto) function

$$v : VAR \longrightarrow Dom$$

where in particular

$$v : AGT \longrightarrow Agt^{\mathbf{n}}$$

too is surjective, with the further requirement that if $x, y \in i^{-1}_{AGT}(k)$ for some $k \in I$, then $v(x) = v(y)$ and if $x \in VAR \backslash AGT$ then $v(x) \notin Agt^{\mathbf{n}}$. Where v and v' are valuations in M, and $v(y) = v'(y)$ for all $y \in VAR$, except (possibly) x, v' is called an *x-variant* of v. We will use $t^{w,v}$ to denote the extension of the term t at world w under valuation v given the model specified by the context. That is, where $t \in CON, t^{w,v} = \mathcal{I}(t, w)$ and where $t \in VAR, t^{w,v} = v(t)$.

That constants are chosen to be defined as non-rigid is motivated by the interpretation of first-order epistemic logic found in [6] and [3]. Here, the formula $K_c a = b$ is read 'c knows that a and b are the same/denote the same object'. Giving the constants a rigid interpretation would result in all identity statements being valid, and hence known, in each particular model. Hence agents would always be able to tell all things apart perfectly – no two things would ever be indistinguishable to any agent, and it would not be possible to model scenarios where an agent has lost track of which coffee cup was his.

When using non-rigid constants, a scoping mechanism is often included in order to disambiguate – like the predicate abstraction of [4]. In the present, such is not required as the reading of possibly ambiguous formulas, as $\hat{K}_t\varphi(a)$, have a natural reading in the epistemic interpretation where knowledge operators are taken to be primary over constants in the sense of [4, p.189]. Such a reading further fits with a sequential decomposition of formulas when using the following truth conditions and allows for meaningful constructions of *de re/de dicto* statements as presented in [3].

[5] It worth noticing that we here define constants as being *non-rigid*, i.e. constants are allowed to take different values in different worlds. This affects the validity of certain axioms, as will be mentioned later.

Truth conditions. Where M is a model, $w \in W, \mathcal{I}$ is an interpretation and v a valuation we denote truth of φ/φ being satisfied at w in M under v by $M, w \vDash_v \varphi$, and define truth/satisfaction recursively as follows:

$$\begin{array}{ll} M, w \vDash_v P(t_1, t_2, ..., t_n) & \text{iff } (t_1^{w,v}, t_2^{w,v}, ..., t_n^{w,v}) \in \mathcal{I}(w, P) \\ M, w \vDash_v t_1 = t_2 & \text{iff } (t_1^{w,v}, t_2^{w,v}) \in \mathcal{I}(=, w) \text{ iff } t_1^{w,v} = t_2^{w,v} \\ M, w \vDash_v \neg\varphi & \text{iff not } M, w \vDash_v \varphi \\ M, w \vDash_v \varphi \wedge \psi & \text{iff } M, w \vDash_v \varphi \text{ and } M, w \vDash_v \psi \\ M, w \vDash_v \forall x P(x) & \text{iff } M, w \vDash_{v'} P(x) \text{ for all } x\text{-variants of } v \end{array}$$

Recall that $\sim (t^{w,v}) \in R^{\mathbf{n}} \cup \{W \times W\}$. We define truth of modal statements thus:

$$M, w \vDash_v K_t\varphi \text{ iff } \forall w' : (w, w') \in \sim (t^{w,v}),\ M, w' \vDash_v \varphi$$
$$M, w \vDash_v \hat{K}_t\varphi \text{ iff } \exists w' : (w, w') \in \sim (t^{w,v}) \text{ and } M, w' \vDash_v \varphi$$

These definitions for operator semantics ensures the validity of both the rule of inference *Knowledge Generalization* and the axioms K and *Dual*, all constitutive of standard, normal modal logics.[6]

The Axiom System $\mathbf{K_n}$

In the following, we list the axioms proposed in [10] for a term-modal version of normal modal logic for n agents in a language $\mathcal{L}^{\mathbf{n}}$, which we will here denote $\mathbf{K_n}$. The axiom system is inspired by those for first-order modal logic used in [2,3,7]. The axiomatic system for $\mathbf{K_n}$ includes all substitution instances of validities of propositional logic[7] and the following axioms (axiom schemes), starting with first-order axioms:

- $\forall$: Where φ is any formula of $\mathcal{L}^{\mathbf{n}}$ and y is a variable not bound in φ: $\forall x\varphi \rightarrow \varphi(y/x)$
- Id: Where t is any term: $t = t$
- PS: For all variables x, y: $(x = y) \rightarrow (\varphi(x) \leftrightarrow \varphi(y))$
- $\exists$Id: For any constant c: $(c = c) \rightarrow \exists x (x = c)$

We must restrict both $\forall$ and PS to variables only since unrestricted versions of these two axioms result in an unsound system when the semantics are defined using non-rigid constants.[8] Further, we include the following modal axioms:

- K: $K_t(\varphi \rightarrow \psi) \rightarrow (K_t\varphi \rightarrow K_t\psi)$
- Dual: $K_t\varphi \leftrightarrow \neg\hat{K}_t\neg\varphi$

Both K and Dual are constitutive axioms of normal modal logics. Further, where $K_x\varphi \in \mathsf{K}_A$, we add the following axioms schemes in order for these modalities be global:

[6] Proofs are left as an exercise to the reader to look up in [10].

[7] That is, where φ is a validity of propositional logic with all propositional variable uniformly replaced by formulas of $\mathcal{L}^{\mathbf{n}}$, φ is an axiom of $\mathbf{K_n}$.

[8] See [3], p. 88-90, for a proof with respect to regular first-order modal logic. The proof given there carries over almost without change.

- T: $K_x\varphi \to \varphi$
- 4: $K_x\varphi \to K_xK_x\varphi$
- B: $\varphi \to K_x\hat{K}_x\varphi$
- $\hat{K}_y\varphi \to \hat{K}_x\varphi$

Here, T, 4 and B in conjunction results in all non-agent objects' accessibility relations being equivalence relations, and Inclusion ensures that all relevant states are related, why the relation(s) are universal. Finally, we include mixed axioms to control the interplay between quantifiers and operators:

- Where $t \neq x$, the Barcan Formula[9]: $\forall x K_t\varphi(x) \to K_t\forall x\varphi(x)$
- Knowledge of Non-identity[10]:$(x_1 \neq x_2) \to K_t(x_1 \neq x_2)$

The inference rules of $\mathbf{K_n}$ consists of Modus Ponens

$$\frac{\varphi, \varphi \to \psi}{\psi}, \tag{MP}$$

and Universal Generalization well-known from first-order logic: where x does not occur free in φ,

$$\frac{\varphi \to \psi}{\varphi \to \forall x\psi} \tag{Gen}$$

and finally Knowledge Generalization constitutive of normal modal logics:

$$\frac{\varphi}{K_t\varphi} \tag{KG}$$

That KG continues to preserve truth under the semantics provided is due to the choice that all non-agent elements of the domain are correlated with total binary relation on W. Had we instead chosen that $K_t\varphi$ should be false for all non-agent denoting terms t and all formulas, as might have been a more just modeling as no non-agent objects ever know anything, the rule would obviously need side-conditions in order to preserve truth. One way to remedy this would be to add agent axioms, $A(x)$, for all the agent variables of AGT, and require that both $A(x)$ and φ be provable before one could conclude $K_x\varphi$. One could then remedy the semantics be requiring that $A(x)$ be true at w in conjunction with our ordinary requirements, but this would result in a conjunction being the main connective in the truth definition for such modal operators, which would then again result in the invalidity of the axiom Dual.

Assuming standard definition of $\mathbf{K_n}$-proofs (a la [1]), we can now define a *normal n agent term-modal logic* as any set of formulas Λ from $\mathcal{L}^{\mathbf{n}}$ that contains all $\mathbf{K_n}$ axioms and which is closed under $\mathbf{K_n}$ inference rules. If we further adopt regular definitions for validity and semantic consequence, say that a logic Λ is *(strongly) complete* with respect to S if, for any set of formulas $\Gamma \cup \{\varphi\}$, if $\Gamma \vDash_{\mathsf{S}} \varphi$, then $\Gamma \vdash_{\Lambda} \varphi$, where S is a class of frames (or models), the following result can be proven:

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9 The BF is not valid if we allow $t = x$. Fortunately, when BF is used in the proof of the existence lemma given in [10], this does not matter.

10 The axiom is restricted to variables as an unrestricted version is invalid with non-rigid constants.

Theorem: Canonical Class Theorem. Any normal n agent term-modal logic Λ is (strongly) complete with respect to the class of canonical models for Λ. ■[11]

Adding Classic Epistemic Axioms

As the construction of the logics presented here have epistemic applications in mind, further axioms are required for the standard interpretation, namely those of classic epistemic logic – T, $K_i\varphi \to \varphi$, and 5, $\neg K_i\varphi \to K_i\neg K_i\varphi$, classically for any $i \in I$, which for validity requires that the accessibility relations of the agents are reflexive for T and euclidean for 5.[12]

When using term-modal operators, even when the accessibility is assumed to have the appropriate properties, this does not entail the validity of axioms with nested operators, such as 4 or 5 – that is, if these are added for all terms.

We can illustrate this with the very simple case of 4, $K_i\varphi \to K_iK_i\varphi$, which when using regular operators and the corresponding semantics, characterize transitivity. For assume for some model M, state w, valuation v, constant a and formula φ that $M, w \models_v K_a\varphi$. Hence, for all w' such that $(w, w') \in\sim (\mathcal{I}(a, w))$, $M, w' \models_v \varphi$. Now assume $M, w \models_v \neg K_aK_a\varphi$. Then for some w'' such that $(w, w'') \in\sim (\mathcal{I}(a, w))$, $M, w'' \models_v \neg K_a\varphi$, and for this we must have for some w''' such that $(w'', w''') \in\sim (\mathcal{I}(a, w''))$ and $M, w''' \models_v \neg\varphi$. Now we may ordinarily conclude by transitivity that as $(w, w''), (w'', w''') \in\sim_i$, so will (w, w''') and hence $M, w''' \models_v \varphi$ by assumption, which leads to a contradiction. This last step is unwarranted, though, under the present semantics as $\sim (\mathcal{I}(a, w))$ and $\sim (\mathcal{I}(a, w''))$ may in fact not be the same relation over W at all, as the interpretation of a may vary form world to world, why there is no guarantee that $\sim (\mathcal{I}(a, w)) =\sim (\mathcal{I}(a, w''))$.

If we on the other hand add such axioms restricted to variables occurring as indexes, these will be valid. That is, if we add $K_x\varphi \to K_xK_x\varphi$, this will be valid exactly on transitive frames, and $\neg K_x\varphi \to K_x\neg K_x\varphi$ will be valid exactly on euclidean frames. Hence adding such versions of T and 5 for all variables will result in a system reminiscent of classic epistemic logic. Here, each agent's accessibility relation will be an equivalence relation, and the accessibility relation of all non-agents will be the universal relation on the frame of the given model. As mentioned, such a system will not validate constant-indexed versions of for example 4.

It is interesting to note that the invalidity of this was thought of as reasonable in [6], where Hintikka writes: "we must therefore assume that the person referred to by a knows that he is referred to by it ... that it is true to say "a knows that he is a"...". For if we assume for some model M that $M, w \models_v \exists x K_a (x = a)$, i.e. that a knows who a is in the reading of [6] and [3], it follows that $M, w \models_v K_a\varphi \to K_aK_a\varphi$. Indeed, it is seen that this results exactly as the assumptions insures that $\mathcal{I}(a, w')$ remains constant for all w' in $\sim (\mathcal{I}(a, w))$, i.e. that the constant a designates rigidly over the states accessible for agent $\mathcal{I}(a, w)$ from w.

[11] For all definitions and the proof of this theorem, see [10].

[12] As T and 5 characterizes reflexive and euclidean frames, respectively, cf. [1].

Examples of Expressibility

Returning to the motivating example from the introduction, we are now in a position to express the student's predicament at the summer school, for $K_a\left(PC\left(x\right) \rightarrow K_x\varphi\right)$ will in fact be a well-formed formula. Hence the student of the example gain the ability to infer from strictly extensional features to second-order information in non-*ad hoc* manner.

Relating to the reasoning of Descartes, the knowledge operators now also function as predicates, hence allowing us to conclude existence from knowledge. Not only is $K_a\varphi \rightarrow \exists x K_x\varphi$ expressible in the present term-modal logic, it is also a validity.

Further, we gain the expressibility to define an "everybody knows that"-operator, E_G, using a unary predicate G to denote the group of agents of interest: we can simply define $E_G\varphi$ as $\forall x\left(G\left(x\right) \rightarrow K_x\varphi\right)$. This is sharp contrast to classic first-order epistemic logic, where $E_G\varphi$ is not well-formed for predicate G from the first-order language.

Moreover, $E_G\varphi$ is true just in the case where all agents in G knows φ – that is, it captures exactly the behavior of the E_G-operator of [3], though it cannot be nested as easily. This is due to the fact that predicates non-rigid, and operators for varying groups would result, cf. [8]. The behavior of rigid, nested operators can be captured if we let $\forall^n x$ denote the quantifier block $\forall x_n \forall x_{n-1} ... \forall x_1$, $G^n\left(x\right)$ denote the conjunction $G\left(x_n\right) \wedge G\left(x_{n-1}\right) \wedge ... \wedge G\left(x_1\right)$ and $K_x^n\varphi$ denote the formula $K_{x_n}K_{x_{n-1}}...K_{x_1}\varphi$.[13] We can then define a series of operators by setting $E_G^0\varphi := \varphi$ and $E_G^{n+1}\varphi := \forall x_{n+1}\forall^n x\left(G^n\left(x\right) \wedge G\left(x_{n+1}\right) \rightarrow K_{x_{n+1}}K_x^n\varphi\right)$. This will result in fixing G throug relevant worlds. We can then define the truth conditions for a 'common knowledge in G' operator, C_G, by $M, w \models_v C_G\varphi$ iff $M, w \models_v E_G^k\varphi$ for $k = 1, 2, ...$

Utilizing the newly-gained expressibility, we can further ensure that membership of G is common knowledge in G and not know by anyone else by requiring the truth of $C_G\forall y\left(G\left(y\right) \leftrightarrow K_{x_1}G\left(y\right)\right)$,[14] and adding to this define "secret club" common knowledge of φ by $C_G\forall y\left(K_y\varphi \leftrightarrow G\left(y\right)\right)$, knowledge that would be handy to be able to express when modeling convention-based choice in coordination games in the style of [5].

Conclusion and Further Perspectives

The system $\mathbf{K_n}$and corresponding semantics presented is behaves nicely with respect completeness, and it is easy to add axioms ensuring the regular properties of first-order epistemic logic, where the resulting systems have an interesting added expressibility. The systems allow for reasoning about knowledge as a property of existing agents, which in turn results in interesting interplay between terms and predicates from the first-order language and the modal operators.

It would be interesting to investigate whether C_G-operator in fact behaves like that of [3], and it is an open question what relation the proposed system

[13] We let both $\forall^0 x$ and $G^0\left(x\right)$ denote the empty string, and $K_n^0\varphi := \varphi$

[14] Where x_1 is bound by the universal quantifier of E_G^1.

stands in with other term-modal systems, like those of [8,11], and to what degree meta-theoretical results can be obtained for varying domains and agent sets for the currently specified systems. Further, to eliminate truth of $K_t\varphi$ where t is a non-agent denoting term, one might consider using a two-sorted language, as proposed in[8]. It would interesting to see what meta-theoretical results could be proven, as well as consider the philosophical foundations for systems in which agents can always tell agents non-agent objects.

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